CONVECTIVE DIFFUSION IN MASS BUBBLING

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A model of spherical cells with a Happel free boundary is used to solve the problem of convective diffusion in mass bubbling. The solutions obtained differ from those already published in respect of a factor which allows for the restricted character of the motion of the bubbles.

In order to describe the process of convective diffusion in mass bubbling we used the model of spherical cells proposed in [1-9]. In this model, the bubbling layer is represented in the form of an isotropic system of uniformly-disposed spherical bubbles of equivalent radius $b = a/\sqrt[3]{\varphi}$. It is well known [10, 11] that, when a single bubble surfaces, a large part of the surface of the bubble is occupied by potential flow, and only in the rear is there a small region of detachment, behind which is a turbulent wake of liquid. The dimensions of the break-off angle have been estimated [10, 12] as being of the order of 1/Re. For bubbles of radius 1.5-2 mm the Reynolds number lies in the range 600-800, and thus the region of detachment is extremely small compared with the total surface area of the bubble. When considering the process of convective diffusion relating to the bubble we therefore took no account of the region of turbulent wake.

For the case of potential flow the velocity field of the liquid is described by the following equations within the framework of the model under consideration [13]:

$$v_r = \frac{U}{1-\varphi}\left[\varphi - \frac{a^3}{r^3}\right]\cos\theta, \tag{1}$$

$$v_\theta = -\frac{U}{1-\varphi}\left[\varphi + \frac{a^3}{2r^3}\right]\sin\theta. \tag{2}$$

For $r = a$

$$v_r = -U\cos\theta;\quad v_\theta = -\frac{U(1+2\varphi)}{2(1-\varphi)}\sin\theta. \tag{3}$$

For $r = a/\sqrt[3]{\varphi}$

$$v_r = 0;\quad v_\theta = -\frac{3}{2}\frac{U\varphi}{(1-\varphi)}\sin\theta. \tag{4}$$

On solving the equation of convective diffusion in the boundary layer of the liquid at the surface of the bubble, subject to the boundary conditions (3), (4) and the boundary-layer velocity-distribution field described by (1), (2), we obtain the following equation for diffusive flow over the whole surface of the bubble:

$$I = 4\sqrt{\frac{2\pi}{3}}\, a^{3/2}D^{1/2}U^{1/2}(c_1 - c_0)\sqrt{\frac{1+2\varphi}{1-\varphi}}. \tag{5}$$

The Nusselt diffusion number

$$\mathrm{Nu} = \frac{Ia}{4\pi a^2(c_1 - c_0)D} = \sqrt{\frac{2(1+2\varphi)}{3\pi(1-\varphi)}}\,\mathrm{Pe}^{1/2}. \tag{6}$$

Equations (5), (6) differ from the corresponding equations for a single bubble derived by V. G. Levich [10] in respect of the factor $\sqrt{(1 + 2\varphi/1 - \varphi)}$, which allows for the restricted motion of the bubbles; the two sets of equations agree as $\varphi \to 0$.

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In practice, the motion of bubbles in a layer of liquid often takes place while the liquid is being vigorously agitated by means of a stirrer, or by the very system of bubbles itself in passing through the liquid. The vigorous agitation of the liquid leads to the establishment of developed turbulence within it. In this case the relative velocity of the bubble has to be taken as the difference between the velocity vector of the bubble ($\vec{v}_1$) and the velocity vector of the liquid ($\vec{v}_0$) at the site of the bubble. The balance of forces acting on the bubbles partly carried along by the liquid takes the form [10]

$$\frac{1}{2}(\rho_l+2\rho_g)V\frac{d\vec{U}}{dt}=V(\rho_l-\rho_g)\frac{d\vec{v}_0}{dt}+F. \tag{7}$$

Equation (7) allows for the augmented mass of liquid in a volume equal to half the volume of the bubble.

The force resisting the motion of the bubble in the case of small bubbles may be calculated from the Marrucci formula [6] obtained by using the model of spherical cells:

$$F_1=12\pi\mu Ua\frac{1-\varphi^{5/3}}{(1-\varphi)^2}. \tag{8}$$

For large bubbles the resistive force

$$F_2=K_f\frac{\rho_l U^2\pi a^2}{2}. \tag{9}$$

Solving Eq. (7) for U by a method analogous to that described in [9] we obtain

$$U=A_2\frac{wa^{1/3}}{\varphi H^{1/3}}.$$

In the case of small bubbles

$$U=\frac{a\varepsilon^{1/2}(1-\varphi)}{12\mu^{1/2}(1-\varphi^{5/3})^{1/2}}. \tag{10}$$

The rate of energy dissipation by turbulent pulsations ε is in order of magnitude [14] equal to

$$\varepsilon=\rho_l\frac{(\Delta U)^3}{l}, \tag{11}$$

where ΔU is the change in the velocity over a distance equal to the scale of the pulsation l.

For turbulent motion in a bubbling layer of small thickness, the maximum scale of the turbulent pulsations will coincide with the height of the foam on the plate H, while the change taking place in the velocity over this distance will coincide with the maximum velocity of the liquid in the bubbling layer, in order of magnitude equal to w/φ. Substituting the value of ε, ΔU, and l into (10) we obtain

$$U=A_1\frac{w^{3/2}(1-\varphi)a}{\sqrt{H\nu}\,\varphi^{3/2}\sqrt{1-\varphi^{5/3}}}. \tag{12}$$

For large bubbles, analogous calculations give

$$U=A_2\frac{wa^{1/3}}{\varphi H^{1/3}}. \tag{13}$$

Substituting (12) and (13) into (5) and (6), we shall have the following for small bubbles

$$I=4\sqrt{\frac{2\pi A_1}{3}}\frac{w^{3/4}a^2D^{1/2}(c_1-c_0)\sqrt{1+2\varphi}}{\varphi^{3/4}\sqrt[4]{H\nu(1-\varphi^{5/3})}}, \tag{14}$$

$$\mathrm{Nu}=\sqrt{\frac{2A_1}{3\pi}}\frac{a}{H}(\mathrm{Re}^*)^{3/4}\mathrm{Pr}^{1/2}\sqrt[4]{\frac{(1+2\varphi)^2}{1-\varphi^{5/3}}}, \tag{15}$$

and for large bubbles

$$I=4\sqrt{\frac{2\pi A_2}{3}}a^{5/3}D^{1/2}\frac{w^{1/2}}{\varphi^{1/2}H^{1/6}}\sqrt{\frac{1+2\varphi}{1-\varphi}}(c_1-c_0), \tag{16}$$

$$\mathrm{Nu} = \sqrt{\frac{2A_2}{3\pi}} \left(\frac{a}{H}\right)^{2/3} (\mathrm{Pe}^*)^{1/2} \sqrt{\frac{1+2\varphi}{1-\varphi}}.$$

It should be noted that, in deriving Eqs. (16) and (17), we took no account of the dependence of the resistance coefficient K_f on the gas content. It is well known [15, 16] that the rate of surfacing of the bubbles increases in mass bubbling, i.e., the resistance coefficient K_f should diminish with increasing φ. However, the actual character of this relationship is still unknown. Hence Eqs. (16) and (17) may be considered as simply a first approximation.

An estimate of the correction to be introduced into the equation of mass transfer for mass bubbling shows that for the gas contents of the layer of foam usually observed (0.5 to 0.8) the correcting factor attached to the coefficient of mass transfer varies from 1.56 to 2.16 in the case of small bubbles and from 2 to 3.6 in the case of large bubbles, i.e., over a fairly considerable range. Equation (14) agrees closely with the correlation derived experimentally in [17], according to which the mass-transfer coefficient in the liquid phase (sulfite–sulfate system, small bubbles) is proportional to the factor $(\varphi/1-\varphi)^{0.19}$. Calculations show that this factor is proportional to the factor $\sqrt[4]{[(1+2\varphi)^2/(1-\varphi^{5/3})]}$.

NOTATION

U	is the relative velocity of the bubble;
v_r and v_θ	are the radial and tangential components of the velocity of the liquid;
r	is the current radius;
θ	is the polar angle;
a	is the radius of the bubble;
b	is the radius of the spherical cell;
φ	is the gas content of the layer of foam;
D	is the coefficient of molecular diffusion;
c_1	is the concentration of absorbed material in the liquid on the surface of the bubble;
c_0	is the concentration of absorbed material in the liquid at the boundary of the spherical cell;
$\vec{v}_1$	is the velocity vector of the bubble;
$\vec{v}_0$	is the velocity vector of the liquid at the site of the bubble;
V	is the volume of the bubble, $V = (4/3)\pi a^3$;
ρ_l, ρ_g	are the densities of the liquid and gas respectively;
F	is the force resisting the motion of the bubble;
K_f	is the resistance coefficient;
μ, ν	are the dynamic and kinematic viscosities of the liquid respectively;
ε	is the rate of energy dissipation by turbulent pulsations;
l	is the scale of the pulsations;
w	is the gas velocity in the complete cross section of the column;
H	is the height of the foam on the plate;
Re	is the Reynolds number, $\mathrm{Re} = Ua/\nu$;
Pr	is the diffusive Prandtl number, $\mathrm{Pr} = \nu/D$;
Pe	is the diffusive Peclet number, $\mathrm{Pe} = Ua/D$;
Nu	is the diffusive Nusselt number, $\mathrm{Nu} = Ia/4\pi a^2 D(c_1 - c_0)$;
I	is the total diffusive flux for a single bubble;
Re*	is the Reynolds number for the whole liquid, $\mathrm{Re}^* = wH/\varphi\nu$;
Pe*	is the Peclet number for the whole liquid, $\mathrm{Pe}^* = wH/\varphi D$.

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